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Corrigendum

Corrigendum to “A Schneider type theorem for Hopf algebroids” [J. Algebra 318 (1) (2007) 225–269]

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ABSTRACT

In our paper we heavily used the result that two constituent bialgebroids in a Hopf algebroid possess isomorphic comodule categories. This statement was based on [T. Brzeziński, A note on coring extensions, Ann. Univ. Ferrara Sez. VII Sci. Mat. LI (2005) 15–27. A corrected version is available at http://arxiv.org/abs/math/0410020v3, Theorem 2.6], whose proof turned out to contain an unjustified step. Here we prove the main results in our paper without using [T. Brzeziński, A note on coring extensions, Ann. Univ. Ferrara Sez. VII Sci. Mat. LI (2005) 15–27. A corrected version is available at http://arxiv.org/abs/math/0410020v3, Theorem 2.6] and the derived isomorphism of comodule categories.

Introduction

Based on [Brz3, Theorem 2.6], it was claimed in [BB2, Theorem 2.2] that the constituent left and right bialgebroids in a Hopf algebroid possess isomorphic comodule categories. Our paper was written with this knowledge. However, recently it turned out that the proof of [Brz3, Theorem 2.6] contains an unjustified step, hence [BB2, Theorem 2.2] is not proven either. There is a similar error also in [Bö2, Proposition 3.1]. Although it has to be stressed that we are not aware of any counterexamples for any of [Brz3, Theorem 2.6] or [BB2, Theorem 2.2], the aim of this corrigendum is to show that, using

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a proper notion of a comodule of a Hopf algebroid, that may differ in general from a comodule of either constituent bialgebroid, the most important results in our paper can be formulated and proven without referring to the objected statements in [Brz3] and [BB2].

Necessary corrections affect some parts of the Appendix, Proposition 3.1, Theorem 4.1, Proposition 4.2, Theorems 5.7, 5.8 and Corollary 6.6 in the paper and also some results used to derive these main claims. Below we go through the listed results and prove their corrected versions. No parts of Section 2 need to be modified.

Throughout, $\mathcal{H}$ is a Hopf algebroid over base algebras L and R, with structure maps denoted as in Appendix A.8.

Corrections to Appendix

In Appendix A.3, [Brz3, Theorem 2.6] is recalled. It is not known to be true without further (purity) assumptions, see the arXiv version of [Brz3]. Namely, the functor $\mathbb{R}$ in (3.1) is known to exist only under the further assumption that the equalizer

$$M \xrightarrow{\quad \varrho^M \quad} M \otimes_A \mathcal{C} \underset{M\otimes_A \Delta_{\mathcal{C}}}{\overset{\varrho^M \otimes_A \mathcal{C}}{\rightrightarrows}} M \otimes_A \mathcal{C} \otimes_A \mathcal{C} \tag{1}$$

in $\mathfrak{M}_L$ is $\mathcal{D} \otimes_L \mathcal{D}$-pure, i.e. it is preserved by the functor $- \otimes_L \mathcal{D} \otimes_L \mathcal{D} : \mathfrak{M}_L \to \mathfrak{M}_L$, for any right $\mathcal{C}$-comodule (M, ϱ^M). In this case we say that $\mathcal{D}$ is a *pure* coring extension of $\mathcal{C}$.

This purity condition holds in several cases that are relevant for our purposes.

Example 1. Any coring extension arising from an L-entwining structure $(A, \mathcal{D}, \psi)$ (in the way described in Appendix A.4) is pure. Indeed, (1) is a split equalizer in $\mathfrak{M}_A$ (split by the right A-module map $M \otimes_A \mathcal{C} \otimes_A \varepsilon_{\mathcal{C}}$). Thus the existence of a forgetful functor $\mathfrak{M}_A \to \mathfrak{M}_L$ implies that (1) is a split equalizer in $\mathfrak{M}_L$, hence it is preserved by any functor of domain $\mathfrak{M}_L$.

Example 2. Any right coring extension of a coseparable A-coring $\mathcal{C}$ is pure. In order to see that, use again that (1) is a split equalizer in $\mathfrak{M}_A$. By separability of the functor $\mathfrak{M}^{\mathcal{C}} \to \mathfrak{M}_A$, it is a split equalizer also in $\mathfrak{M}^{\mathcal{C}}$. If $\mathcal{D}$ is an L-coring that is a right extension of $\mathcal{C}$, then taking cotensor products with the $\mathcal{C}$-$\mathcal{D}$ bicomodule $\mathcal{C}$ defines a functor $-\Box_{\mathcal{C}}\mathcal{C} : \mathfrak{M}^{\mathcal{C}} \to \mathfrak{M}_L$, equipping any right $\mathcal{C}$-comodule $M \cong M\Box_{\mathcal{C}}\mathcal{C}$ with a right L-action. By right L-linearity of any $\mathcal{C}$-comodule map, splitting of the equalizer (1) in $\mathfrak{M}^{\mathcal{C}}$ implies that it splits in also in $\mathfrak{M}_L$. Hence the purity condition holds.

In Appendix A.9, [BB2, Theorem 2.2] is recalled. Since it is based on [Brz3, Theorem 2.6], it is not known to hold. This means that existence of the isomorphism functors $\mathbb{R}$, $\widetilde{\mathbb{R}}$, $\mathbb{L}$ and $\widetilde{\mathbb{L}}$, on page 265, in (A.17) and in (A.22) is not justified. In what follows we provide some results substituting the unjustified claims in Appendix A.9.

The following definition was proposed in [Bö2, Definition 3.2] and [BaSz, Section 2.2].

Definition 3. A *right comodule* of a Hopf algebroid $\mathcal{H}$ is a right L-module as well as a right R-module M, together with a right coaction $\varrho_R : M \to M \otimes_R H$ of the constituent right bialgebroid $\mathcal{H}_R$ and a right coaction $\varrho_L : M \to M \otimes_L H$ of the constituent left bialgebroid $\mathcal{H}_L$, such that ϱ_R is an $\mathcal{H}_L$-comodule map and ϱ_L is an $\mathcal{H}_R$-comodule map. Explicitly, ϱ_R is right L-linear, ϱ_L is right R-linear and

$$(M \otimes_R \gamma_L) \circ \varrho_R = (\varrho_R \otimes_L H) \circ \varrho_L \quad \text{and} \quad (M \otimes_L \gamma_R) \circ \varrho_L = (\varrho_L \otimes_R H) \circ \varrho_R. \tag{2}$$

Morphisms of $\mathcal{H}$-comodules are $\mathcal{H}_R$-comodule maps as well as $\mathcal{H}_L$-comodule maps. The category of right $\mathcal{H}$-comodules is denoted by $\mathfrak{M}^{\mathcal{H}}$.

The category ${}^{\mathcal{H}}\mathfrak{M}$ of left $\mathcal{H}$-comodules is defined symmetrically.

Since a comodule M of a Hopf algebroid $\mathcal{H}$ is a comodule of both constituent bialgebroids $\mathcal{H}_L$ and $\mathcal{H}_R$, we can consider the coinvariants $M^{co\,\mathcal{H}_R}$ and $M^{co\,\mathcal{H}_L}$ in the sense of Appendix A.6.

Proposition 4. *(See [BB2, Corrigendum].) Consider a Hopf algebroid $\mathcal{H}$ and a right $\mathcal{H}$-comodule M. Then $M^{co\,\mathcal{H}_R} \subseteq M^{co\,\mathcal{H}_L}$. If the antipode of $\mathcal{H}$ is bijective then an equality holds.*

Proposition 5. *For any Hopf algebroid $\mathcal{H}$ the following hold.*

(1) *The forgetful functor $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}_L$ possesses a right adjoint $- \otimes_L H$.*
(2) *The forgetful functor $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}_R$ possesses a right adjoint $- \otimes_R H$.*

Proof. (1) The unit of the adjunction is given by the $\mathcal{H}_L$-coaction $M \to M \otimes_L H$, for any right $\mathcal{H}$-comodule M. It is an $\mathcal{H}$-comodule map by definition. Counit is given by $N \otimes_L \pi_L : N \otimes_L H \to N$, for any right L-module N. Part (2) is proven symmetrically. $\square$

Theorem 6. *(See [BB2, Corrigendum].) Consider a Hopf algebroid $\mathcal{H}$. Denote by F_R and F_L the forgetful functors $\mathfrak{M}^{\mathcal{H}_R} \to \mathfrak{M}_k$ and $\mathfrak{M}^{\mathcal{H}_L} \to \mathfrak{M}_k$, respectively.*

(1) *If the equalizer*

$$M \xrightarrow{\varrho_R} M \otimes_R H \overset{\varrho_R \otimes_R H}{\underset{M \otimes_R \gamma_R}{\rightrightarrows}} M \otimes_R H \otimes_R H \tag{3}$$

in $\mathfrak{M}_L$ is $H \otimes_L H$-pure, i.e. it is preserved by the functor $- \otimes_L H \otimes_L H : \mathfrak{M}_L \to \mathfrak{M}_L$, for any right $\mathcal{H}_R$-comodule (M, ϱ_R), then there exists a functor $U : \mathfrak{M}^{\mathcal{H}_R} \to \mathfrak{M}^{\mathcal{H}_L}$, such that $F_L \circ U = F_R$. Moreover, in this case the forgetful functor $G_R : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$ is fully faithful.

(2) *If the equalizer*

$$N \xrightarrow{\varrho_L} N \otimes_L H \overset{\varrho_L \otimes_L H}{\underset{N \otimes_L \gamma_L}{\rightrightarrows}} N \otimes_L H \otimes_L H$$

in $\mathfrak{M}_R$ is $H \otimes_R H$-pure, i.e. it is preserved by the functor $- \otimes_R H \otimes_R H : \mathfrak{M}_R \to \mathfrak{M}_R$, for any right $\mathcal{H}_L$-comodule (N, ϱ_L), then there exists a functor $V : \mathfrak{M}^{\mathcal{H}_L} \to \mathfrak{M}^{\mathcal{H}_R}$, such that $F_R \circ V = F_L$. Moreover, in this case the forgetful functor $G_L : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_L}$ is fully faithful.

(3) *If both purity assumptions in parts (1) and (2) hold, then the forgetful functors $G_R : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$ and $G_L : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_L}$ are isomorphisms. Moreover, $G_L \circ G_R^{-1} = U$ and $G_R \circ G_L^{-1} = V$, hence U and V are inverse isomorphisms.*

Theorem 7. *(See [BB2, Corrigendum].) For any Hopf algebroid $\mathcal{H}$, the category $\mathfrak{M}^{\mathcal{H}}$ of right $\mathcal{H}$-comodules is monoidal. Moreover, the following diagram is commutative and all occurring forgetful functors are strict monoidal.*

$$\begin{tikzcd}
\mathfrak{M}^{\mathcal{H}} \arrow[r, "G_R"] \arrow[d, "G_L"'] & \mathfrak{M}^{\mathcal{H}_R} \arrow[d] \\
\mathfrak{M}^{\mathcal{H}_L} \arrow[r] & {}_R\mathfrak{M}_R.
\end{tikzcd}$$

Definition 8. A *right comodule algebra* of a Hopf algebroid $\mathcal{H}$ is an algebra in the monoidal category $\mathfrak{M}^{\mathcal{H}}$. Right/left modules of a right $\mathcal{H}$-comodule algebra A *in* $\mathfrak{M}^{\mathcal{H}}$ are termed (*right–right/left–right*) *relative Hopf modules*. Their categories are denoted by $\mathfrak{M}_A^{\mathcal{H}}$ and ${}_A\mathfrak{M}^{\mathcal{H}}$, respectively.

Remark 9. The antipode S of a Hopf algebroid induces strict anti-monoidal functors ${}^{\mathcal{H}_R}\mathfrak{M} \to \mathfrak{M}^{\mathcal{H}_L}$, ${}^{\mathcal{H}_L}\mathfrak{M} \to \mathfrak{M}^{\mathcal{H}_R}$ and ${}^{\mathcal{H}}\mathfrak{M} \to \mathfrak{M}^{\mathcal{H}}$ as follows. Let M be a left $\mathcal{H}_R$-comodule with coaction $m \mapsto m^{[-1]} \otimes_R m^{[0]}$. Then M has a right $\mathcal{H}_L$-comodule structure with right L-action $ml := \pi_R \circ t_L(l)m$, for $l \in L$ and $m \in M$, and coaction

$$m \mapsto m^{[0]} \underset{L}{\otimes} S\big(m^{[-1]}\big). \tag{4}$$

If M is a left $\mathcal{H}_L$-comodule with coaction $m \mapsto m_{[-1]} \otimes_L m_{[0]}$, then M has a right $\mathcal{H}_R$-comodule structure, with right R-action $mr := \pi_L \circ t_R(r)m$, for $r \in R$ and $m \in M$, and coaction

$$m \mapsto m_{[0]} \underset{R}{\otimes} S(m_{[-1]}). \tag{5}$$

If M is a left $\mathcal{H}$-comodule then the $\mathcal{H}_R$-coaction (5) and the $\mathcal{H}_L$-coaction (4) are checked to constitute a right $\mathcal{H}$-comodule structure on M.

Clearly, if S is bijective, then all these functors are isomorphisms. Therefore, A is a right $\mathcal{H}$-comodule algebra if and only if the opposite algebra A^{op} possesses a left $\mathcal{H}$-comodule algebra structure.

Definition 10. For a right comodule algebra A of a Hopf algebroid $\mathcal{H}$ with a bijective antipode, consider the left $\mathcal{H}$-comodule algebra A^{op} as in Remark 9. Left/right A^{op}-modules in ${}^{\mathcal{H}}\mathfrak{M}$ are called (*right–left/left–left*) *relative Hopf modules*. Their categories are denoted by ${}^{\mathcal{H}}\mathfrak{M}_A$ and ${}_A{}^{\mathcal{H}}\mathfrak{M}$, respectively.

Relative Hopf modules in the sense of Definition 8 and Definition 10 are no longer identified with comodules of corings.

Proposition 11. *For a right comodule algebra A of a Hopf algebroid $\mathcal{H}$, set $B := A^{co\,\mathcal{H}_R}$. Then there is an adjunction*

$$- \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}} \qquad (-)^{co\,\mathcal{H}_R} : \mathfrak{M}_A^{\mathcal{H}} \to \mathfrak{M}_B.$$

Proof. For any right B-module N, the unit of the adjunction is given by the right B-module map

$$N \to (N \otimes_B A)^{co\,\mathcal{H}_R}, \qquad n \mapsto n \otimes_B 1_A.$$

For any relative Hopf module $M \in \mathfrak{M}_A^{\mathcal{H}}$, counit is given by

$$M^{co\,\mathcal{H}_R} \otimes_B A \to M, \qquad m \otimes_B a \mapsto ma.$$

Obviously, it is a right A-module map. In light of Proposition 4, it is also a morphism of $\mathcal{H}$-comodules. Verification of the adjunction relations is a routine computation. □

Proposition 12. *For a Hopf algebroid $\mathcal{H}$ and a right $\mathcal{H}$-comodule algebra A, set $B := A^{co\,\mathcal{H}_R}$.*

(1) *The functor $- \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}_R}$ is fully faithful if and only if the functor $- \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}}$ is fully faithful.*
(2) *If the functor $- \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}_R}$ is an equivalence then also the functor $- \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}}$ is an equivalence.*

Proof. Consider the adjunction in Proposition 11 and the adjunction

$$-\otimes_B A : \mathfrak{M}_B \to \mathfrak{M}_A^{\mathcal{H}_R} \qquad (-)^{co\,\mathcal{H}_R} : \mathfrak{M}_A^{\mathcal{H}_R} \to \mathfrak{M}_B. \tag{6}$$

Both statements follow by noting that the units of the two adjunctions coincide and counit of the adjunction in Proposition 11 is equal to the restriction of the counit of the adjunction (6) to the objects of $\mathfrak{M}_A^{\mathcal{H}}$. □

Corrections to Proposition 3.1

The functor $\mathbb{R}$ in (3.1) is known to exist only under the further assumption that the equalizer (1) in $\mathfrak{M}_L$ is $\mathcal{D} \otimes_L \mathcal{D}$-pure, for any right $\mathcal{C}$-comodule (M, ϱ^M). Therefore Proposition 3.1 is justified only under this purity assumption.

After Proposition 3.1 particular coring extensions over equal base algebras are discussed. The correspondence between such coring extensions and coring maps holds only in a more restricted situation, see [BB3, Corrigendum]. Therefore the beginning of the last paragraph on page 238 should be modified as follows.

If the two corings $\mathcal{C}$ and $\mathcal{D}$ are equal and $\mathbb{R}$ is the identity functor, then Proposition 3.1 reduces to [Brz1, Corollary 3.6]. More generally, let $\mathcal{C}$ and $\mathcal{D}$ be corings over the same base algebra A such that the right A-actions of the A-coring $\mathcal{C}$ and the right $\mathcal{D}$-comodule $\mathcal{C}$ coincide. Then $\mathcal{D}$ is a (necessarily pure) right extension of $\mathcal{C}$ if and only if there exists a homomorphism of A-corings $\kappa : \mathcal{C} \to \mathcal{D}$ (in terms of which the $\mathcal{D}$-coaction on $\mathcal{C}$ is given by $\tau_{\mathcal{C}} := (\mathcal{C} \otimes_A \kappa) \circ \Delta_{\mathcal{C}}$), cf. [BB3, Corrigendum].

Since by Example 1 the purity assumption needed in Proposition 3.1 holds whenever the coring $\mathcal{C}$ arises from an L-entwining structure $(A, \mathcal{D}, \psi)$ (in the way described in Appendix A.4), no other results in Section 3 need to be modified.

Corrections to Theorem 4.1

For a right comodule algebra A of Hopf algebroid $\mathcal{H}$, consider the forgetful functors

$$\mathfrak{M}_A^{\mathcal{H}} \xrightarrow{\quad\mathbb{R}\quad} \mathfrak{M}^{\mathcal{H}} \xrightarrow{\quad\mathbb{V}\quad} \mathfrak{M}^{\mathcal{H}_L} \xrightarrow{\quad\mathbb{U}\quad} \mathfrak{M}_L. \tag{7}$$

In our paper the functor $\mathbb{V}$ was believed to be a (trivial) isomorphism and relative separability of $\mathbb{U}$ with respect to $\mathbb{R}$ was related to $\mathcal{I}_{\mathbb{U}}$-injectivity of A. Allowing for $\mathbb{V}$ not to be an isomorphism, one can study relative separability of $\mathbb{U}$ with respect to $\mathbb{VR}$ or relative separability of $\mathbb{UV}$ with respect to $\mathbb{R}$. This results in the two theorems below, replacing Theorem 4.1. Note that none of the functors $\mathbb{R}$ and $\mathbb{VR}$ corresponds to a coring extension, in particular the functor $\mathbb{R} : \mathfrak{M}_A^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}}$ on page 227 has no such interpretation.

Theorem 13. *Consider a Hopf algebroid $\mathcal{H}$, with constituent left bialgebroid $\mathcal{H}_L$, right bialgebroid $\mathcal{H}_R$ and antipode S. For a right $\mathcal{H}$-comodule algebra A, the following assertions are equivalent.*

- (a) *There exists a right total integral in the (bijective) L-entwining structure* (A.18) *with grouplike element 1_H, i.e. a morphism $j \in \mathrm{Hom}^{\mathcal{H}_L}(H, A)$, normalized as $j(1_H) = 1_A$.*
- (b) *$A \in \mathfrak{M}^{\mathcal{H}_L}$ is $\mathcal{I}_{\mathbb{U}}$-injective (i.e. A is a relative injective right $\mathcal{H}_L$-comodule).*
- (c) *Any object in the image of $\mathbb{VR}$ is $\mathcal{I}_{\mathbb{U}}$-injective (i.e. injective with respect to $\mathbb{U}$).*
- (d) *The functor $\mathbb{U}$ is $(\mathfrak{M}^{\mathcal{H}_L}, \mathbb{VR})$-separable.*

If the antipode of $\mathcal{H}$ is bijective then the following statements are also equivalent to the foregoing ones.

- (e) *There exists a left total integral in the bijective R-entwining structure* (A.11) *with grouplike element 1_H, i.e. a left $\mathcal{H}_R$-colinear map $j_{cop}^{op} : H \to A$, normalized as $j_{cop}^{op}(1_H) = 1_A$.*

(f) *A is a relative injective left $\mathcal{H}_R$-comodule.*
(g) *Any object of ${}^{\mathcal{H}}\mathfrak{M}_A$ is a relative injective left $\mathcal{H}_R$-comodule.*
(h) *The forgetful functor ${}^{\mathcal{H}_R}\mathfrak{M} \to {}_R\mathfrak{M}$ is $({}^{\mathcal{H}_R}\mathfrak{M}, (\mathbb{VR})^{op}_{cop})$-relative separable, where $(\mathbb{VR})^{op}_{cop}$ denotes the forgetful functor ${}^{\mathcal{H}}\mathfrak{M}_A \to {}^{\mathcal{H}_R}\mathfrak{M}$.*

If the antipode of $\mathcal{H}$ is bijective then the following statements are also equivalent to each other (*but not necessarily to the foregoing ones*)*.*

(i) *There exists a right total integral in the R-entwining structure* (A.11) *with grouplike element 1_H, i.e. a morphism $j^{op} \in \mathrm{Hom}^{\mathcal{H}_R}(H, A)$, normalized as $j^{op}(1_H) = 1_A$.*
(j) *A is a relative injective right $\mathcal{H}_R$-comodule.*
(k) *Any object of ${}_A\mathfrak{M}^{\mathcal{H}}$ is a relative injective right $\mathcal{H}_R$-comodule.*
(l) *The forgetful functor $\mathfrak{M}^{\mathcal{H}_R} \to \mathfrak{M}_R$ is $(\mathfrak{M}^{\mathcal{H}_R}, (\mathbb{VR})^{op})$-relative separable, where $(\mathbb{VR})^{op}$ denotes the forgetful functor ${}_A\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$.*
(m) *There exists a left total integral in the bijective L-entwining structure* (A.18) *with grouplike element 1_H, i.e. a left $\mathcal{H}_L$-colinear map $j_{cop} : H \to A$, normalized as $j_{cop}(1_H) = 1_A$.*
(n) *A is a relative injective left $\mathcal{H}_L$-comodule.*
(o) *Any object of ${}^{\mathcal{H}}_A\mathfrak{M}$ is a relative injective left $\mathcal{H}_L$-comodule.*
(p) *The forgetful functor ${}^{\mathcal{H}_L}\mathfrak{M} \to {}_L\mathfrak{M}$ is $({}^{\mathcal{H}_L}\mathfrak{M}, (\mathbb{VR})_{cop})$-relative separable, where $(\mathbb{VR})_{cop}$ denotes the forgetful functor ${}^{\mathcal{H}}_A\mathfrak{M} \to {}^{\mathcal{H}}_L\mathfrak{M}$.*

Proof. (a) $\Rightarrow$ (d). In light of Theorem 2.12(3), we need to construct a right $\mathcal{H}_L$-colinear natural retraction ν_M of the $\mathcal{H}_L$-coaction, for any $M \in \mathfrak{M}^{\mathcal{H}}_A$. In terms of the map j in part (a), it is given by the well defined maps

$$\nu_M : M \otimes_L H \to M, \qquad m \otimes_L h \mapsto m^{[0]} j\big(S\big(m^{[1]}\big)h\big), \tag{8}$$

where the Sweedler type index notation $m \mapsto m^{[0]} \otimes_R m^{[1]}$ is used for the $\mathcal{H}_R$-coaction on M.

All other implications are proven by the same steps used to prove Theorem 4.1. However, since the categories $\mathfrak{M}^{\mathcal{H}_L}$ and $\mathfrak{M}^{\mathcal{H}_R}$ may be different, assertions (a) and (i) are not known to be equivalent. $\square$

Theorem 14. *For a right comodule algebra A of a Hopf algebroid $\mathcal{H}$, the following assertions are equivalent.*

(a) *There exists a normalized right $\mathcal{H}$-comodule map $j : H \to A$.*
(b) *The object A in $\mathfrak{M}^{\mathcal{H}}$ is $\mathcal{I}_{\mathbb{UV}}$-injective.*
(c) *Any object in the image of $\mathbb{R}$ is $\mathcal{I}_{\mathbb{UV}}$-injective.*
(d) *The functor $\mathbb{UV}$ is $(\mathfrak{M}^{\mathcal{H}}, \mathbb{R})$-separable.*

If the antipode of $\mathcal{H}$ is bijective, then these equivalent statements are equivalent also to the existence of a normalized left $\mathcal{H}$-comodule map $H \to A$, hence the symmetrical counterparts of (b)–(d)*.*

Proof. (d) $\Rightarrow$ (c) follows by Proposition 5(1) and Corollary 2.9(2).

(c) $\Rightarrow$ (b) is obvious.

(b) $\Rightarrow$ (a) Denote by $\eta : R \to A$ the unit of the R-ring A. Since $\eta \circ \pi_R \circ t_L : L \to A$ and $t_L : L \to H$ are $\mathcal{H}$-comodule maps and t_L is a split monomorphism of right L-modules, using $\mathcal{I}_{\mathbb{UV}}$-injectivity of A, j is constructed as the unique $\mathcal{H}$-comodule map for which $j \circ t_L = \eta \circ \pi_R \circ t_L$.

(a) $\Rightarrow$ (d) We need to construct an $\mathcal{H}$-colinear (i.e. $\mathcal{H}_L$-colinear and $\mathcal{H}_R$-colinear) natural retraction ν_M of the $\mathcal{H}_L$-coaction, for any object M in $\mathfrak{M}^{\mathcal{H}}_A$. In terms of the map j in part (a), it is given by the same formula (8). Since j is an $\mathcal{H}$-comodule map, so in ν_M.

If the antipode is bijective then any (normalized) right $\mathcal{H}$-comodule map $j : H \to A$ determines a (normalized) left $\mathcal{H}$-comodule map $j \circ S^{-1} : H \to A$. This correspondence is clearly bijective. $\square$

Obviously, if the equivalent statements in Theorem 14 hold then also the equivalent statements in Theorem 13 hold.

Corrections to Proposition 4.2

Consider a Hopf algebroid $\mathcal{H}$ with a bijective antipode and a right $\mathcal{H}$-comodule algebra A. Set $B := A^{co\,\mathcal{H}_R} = A^{co\,\mathcal{H}_L}$, cf. Proposition 4. By Appendix A.13, one can associate to A four (anti-) isomorphic corings. Clearly, if any of them is a Galois coring (with respect to the grouplike element determined by the unit elements in A and $\mathcal{H}$), then all of them are Galois corings. In other words, the four properties that $B \subseteq A$ is a left or right Galois extension by $\mathcal{H}_R$ or $\mathcal{H}_L$ are all equivalent to each other. In the corrected version of Proposition 4.2, $\mathcal{H}$-comodule algebras A are studied, such that these equivalent Galois conditions hold.

Lemma 15. *Let $\mathcal{H}$ be a Hopf algebroid with constituent left bialgebroid $\mathcal{H}_L = (H, L, s_L, t_L, \gamma_L, \pi_L)$, right bialgebroid $\mathcal{H}_R = (H, R, s_R, t_R, \gamma_R, \pi_R)$, and a bijective antipode S. Assume that H is a projective right comodule for the R-coring (H, γ_R, π_R) via γ_R. Then H is a projective left L-module via left multiplication by s_L.*

Proof. By [BW, 18.20(1)], projectivity of H as a right $\mathcal{H}_R$-comodule implies that H is a projective right R-module via the action

$$H \otimes R \to H, \qquad h \otimes r \mapsto h s_R(r). \tag{9}$$

By bijectivity of the antipode, the right R-module (9) is isomorphic to the right R-module H, with action

$$H \otimes R \to H, \qquad h \otimes r \mapsto t_R(r) h. \tag{10}$$

Hence also the right R-module (10) is projective. Furthermore, the algebra isomorphism $\pi_R \circ s_L : L^{op} \to R$ induces a category isomorphism $\mathfrak{M}_R \cong {}_L\mathfrak{M}$. This isomorphism takes the projective right R-module (10) to the projective left L-module H, with action

$$L \otimes H \to H, \qquad l \otimes h \mapsto t_R \circ \pi_R \circ s_L(l) h = s_L(l) h. \qquad \square$$

The following replaces Proposition 4.2.

Proposition 16. *Let $\mathcal{H}$ be a Hopf algebroid with constituent left bialgebroid $\mathcal{H}_L = (H, L, s_L, t_L, \gamma_L, \pi_L)$, right bialgebroid $\mathcal{H}_R = (H, R, s_R, t_R, \gamma_R, \pi_R)$, and a bijective antipode S. Assume that H is a projective left R-module via t_R and a projective right comodule for the R-coring (H, γ_R, π_R) via γ_R. (These assumptions hold e.g. if H is finitely generated and projective both as a right and left L-module and also as a right and left R-module, cf.* [Bö2, *Section* 4].) *Then $\mathfrak{M}^{\mathcal{H}_L} \cong \mathfrak{M}^{\mathcal{H}} \cong \mathfrak{M}^{\mathcal{H}_R}$ and ${}^{\mathcal{H}_L}\mathfrak{M} \cong {}^{\mathcal{H}}\mathfrak{M} \cong {}^{\mathcal{H}_R}\mathfrak{M}$ as monoidal categories. Moreover, for a right $\mathcal{H}$-comodule algebra A, such that $B := A^{co\,\mathcal{H}_R} \subseteq A$ is a right $\mathcal{H}_R$-Galois extension, the following assertions are equivalent.*

- (a) *A is a faithfully flat right B-module.*
- (b) *B is a direct summand of the right B-module A.*
- (c) *The functors $A \otimes_B \bullet : {}_B\mathfrak{M} \to {}^{\mathcal{H}}_{A}\mathfrak{M}$ and ${}^{co\,\mathcal{H}}(\bullet) : {}^{\mathcal{H}}_{A}\mathfrak{M} \to {}_B\mathfrak{M}$ are inverse equivalences and $H \otimes_R A$ is a flat right A-module.*
- (d) *A is a projective generator in ${}^{\mathcal{H}}_{A}\mathfrak{M}$ and $H \otimes_R A$ is a flat right A-module.*
- (e) *A is a generator of right B-modules.*
- (f) *A is a faithfully flat left B-module.*
- (g) *B is a direct summand of the left B-module A.*
- (h) *The functors $\bullet \otimes_B A : \mathfrak{M}_B \to \mathfrak{M}^{\mathcal{H}}_{A}$ and $(\bullet)^{co\,\mathcal{H}} : \mathfrak{M}^{\mathcal{H}}_{A} \to \mathfrak{M}_B$ are inverse equivalences.*
- (i) *A is a projective generator in $\mathfrak{M}^{\mathcal{H}}_{A}$.*
- (j) *A is a generator of left B-modules.*
- (k) *The equivalent conditions in Theorem 14 hold.*

Proof. Since H is a projective left R-module by assumption, it is in particular flat. As a left L-module, H is projective hence flat by Lemma 15. Thus the monoidal isomorphisms $\mathfrak{M}^{\mathcal{H}} \cong \mathfrak{M}^{\mathcal{H}} \cong \mathfrak{M}^{\mathcal{H}_R}$ follow by Theorem 6(3) and Theorem 7. Bijectivity of the antipode implies strict anti-monoidal isomorphisms ${}^{\mathcal{H}_R}\mathfrak{M} \cong \mathfrak{M}^{\mathcal{H}_L}$, ${}^{\mathcal{H}_L}\mathfrak{M} \cong \mathfrak{M}^{\mathcal{H}_R}$ and ${}^{\mathcal{H}}\mathfrak{M} \cong \mathfrak{M}^{\mathcal{H}}$, cf. Remark 9. Hence also ${}^{\mathcal{H}_L}\mathfrak{M} \cong {}^{\mathcal{H}}\mathfrak{M} \cong {}^{\mathcal{H}_R}\mathfrak{M}$. Note that this implies in particular ${}^{\mathcal{H}_R}_{A}\mathfrak{M} \cong {}^{\mathcal{H}}_{A}\mathfrak{M}$ and $\mathfrak{M}^{\mathcal{H}_R}_{A} \cong \mathfrak{M}^{\mathcal{H}}_{A}$. In light of these isomorphisms, equivalence of assertions (a)–(k) follows by the same proof in the paper. $\square$

Corrections to Theorem 5.7

The most important result in our paper is a Schneider type Theorem 5.7. Here we show that it is true for a comodule algebra A of a Hopf algebroid $\mathcal{H}$, in the sense of Definition 8.

Lemma 5.2 and Remark 5.4 (as they are formulated) are meaningless, as the functors in Fig. 3.1 are not known to exist. Remark 5.4 needs to be reformulated as follows.

Remark 17. Consider a Hopf algebroid $\mathcal{H}$ and a right $\mathcal{H}$-comodule algebra A. The lifted canonical map (5.3) is a split epimorphism of right L-modules i.e., using the notations in (7), it belongs to $\mathcal{E}_{\mathbb{UVR}}$ in various situations.

(1) If the (right L-linear) canonical map (A.12) is surjective and $A \otimes_R H$ is a projective right L-module. The latter condition holds provided that H is a projective right L-module (via t_L) and A is a projective right R-module.
(2) If the (right A-linear) canonical map (A.12) is surjective, $A \otimes_R H$ is a projective right A-module (e.g. the antipode is bijective and H is a projective right R-module via s_R) and $\mathcal{E}_{\mathbb{U}^{\mathcal{C}}} \subseteq \mathcal{E}_{\mathbb{UVR}}$, where $\mathbb{U}^{\mathcal{C}}$ denotes the forgetful functor $\mathfrak{M}^{\mathcal{H}}_{A} \to \mathfrak{M}_A$.

The condition $\mathcal{E}_{\mathbb{U}^{\mathcal{C}}} \subseteq \mathcal{E}_{\mathbb{UVR}}$ holds whenever dealing with a comodule algebra A of a Hopf algebra $\mathcal{H}$ over a commutative ring k. Indeed, in this case $\mathbb{V}$ is the identity functor $\mathfrak{M}^{\mathcal{H}}$, and the functors $\mathbb{U}^{\mathcal{C}}$, $\mathbb{R}$ and $\mathbb{U}$ are forgetful functors. A fourth forgetful functor $\mathfrak{M}_A \to \mathfrak{M}_k$ makes the following diagram commutative.

$$\begin{array}{ccc} \mathfrak{M}^{\mathcal{H}}_{A} & \xrightarrow{\mathbb{R}} & \mathfrak{M}^{\mathcal{H}} \\ {\scriptstyle \mathbb{U}^{\mathcal{C}}}\downarrow & & \downarrow{\scriptstyle \mathbb{U}} \\ \mathfrak{M}_A & \longrightarrow & \mathfrak{M}_k \end{array}$$

This proves that in this case $\mathcal{E}_{\mathbb{U}^{\mathcal{C}}} \subseteq \mathcal{E}_{\mathbb{UVR}}$, thus assumptions (2) hold e.g. in Schneider's theorem [Schn, Theorem I].

Lemma 18. *Regard a right comodule algebra A of a Hopf algebroid $\mathcal{H}$ as a right $\mathcal{H}_R$-comodule algebra. Then the following associated maps are morphisms in $\mathfrak{M}^{\mathcal{H}}_{A}$.*

(1) *The entwining map in* (A.11);
(2) *The lifted canonical map* (5.3).

Proof. $H \otimes_R A$ is an object in $\mathfrak{M}^{\mathcal{H}}_{A}$ via the A-action induced by the multiplication in A and the diagonal $\mathcal{H}_R$- and $\mathcal{H}_L$-coactions. $A \otimes_R H$ is an object in $\mathfrak{M}^{\mathcal{H}}_{A}$ via the A-action $(a \otimes_R h)a' = aa'^{[0]} \otimes_R ha'^{[1]}$, (where $a \mapsto a^{[0]} \otimes_R a^{[1]}$ denotes the $\mathcal{H}_R$-coaction) and $\mathcal{H}_R$- and $\mathcal{H}_L$-coactions induced by the respective coproducts in $\mathcal{H}$. $A \otimes_T A$ is an object in $\mathfrak{M}^{\mathcal{H}}_{A}$ via the relative Hopf module structure of the second factor. It is left to the reader to check that both maps in the lemma are compatible with these structures. $\square$

Lemma 5.6 is modified as follows.

Lemma 19. *Let $\mathcal{H}$ be a Hopf algebroid with a bijective antipode and let A be a right $\mathcal{H}$-comodule algebra such that the equivalent statements in Theorem* 14 *hold. Take an algebra T such that $B := A^{co\,\mathcal{H}_R}$ is a T-ring. Under these assumptions, if the lifted canonical map* (5.3) *possesses a right L-module section ζ_0^T, then the map ζ^T constructed in Lemma* 5.6 *is a section of* (5.3) *in $\mathfrak{M}_A^{\mathcal{H}}$.*

Proof. Existence of a section in $\mathfrak{M}_A^{\mathcal{H}}$ of the lifted canonical map follows by the same reasoning in the proof of Lemma 5.6: by assumption, the forgetful functor $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}_L$ is relative separable. Hence by Theorem 2.8(1) it reflects split epimorphisms. Since by definition $\mathfrak{M}_A^{\mathcal{H}}$ is the category of modules for the monad $-\otimes_R A : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}}$, the forgetful functor $\mathfrak{M}_A^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}}$ possesses a left adjoint $-\otimes_R A : \mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}_A^{\mathcal{H}}$ (where for any right $\mathcal{H}$-comodule M, $M \otimes_R A$ is a relative Hopf module via the A-action on the second factor and the diagonal coactions). Hence by the same reasoning in Remark 5.3, Lemma 18(1) implies that the lifted canonical map has a section in $\mathfrak{M}_A^{\mathcal{H}}$.

The reference to (unjustified) Proposition 3.1 in the explicit construction of ζ^T can be avoided. By the proof of Theorem 14, the right $\mathcal{H}$-colinear natural retraction ν_M of the $\mathcal{H}_L$-coaction, for any $M \in \mathfrak{M}_A^{\mathcal{H}}$, can be expressed in terms of a normalized right $\mathcal{H}$-comodule map $j : H \to A$ as in (8). Thus the same formula of ζ^T in Lemma 5.6 is obtained. □

Theorem 5.7 and its proof need to be modified as follows.

Theorem 20. *Let $\mathcal{H}$ be a Hopf algebroid with a bijective antipode and let A be a right $\mathcal{H}$-comodule algebra. Put $B := A^{co\,\mathcal{H}_R}$ and take an algebra T such that B is a T-ring. In this setting, if the lifted canonical map* (5.3) *is a split epimorphism of right L-modules, then the following are equivalent.*

(a) *The canonical map* (A.12) *is bijective and the inclusion $B \to A$ splits in $\mathfrak{M}_B$.*
(b) *The canonical map* (A.12) *is bijective and the inclusion $B \to A$ splits in ${}_B\mathfrak{M}$.*
(c) *The equivalent statements in Theorem* 14 *hold.*
(d) *$A \otimes_B - : {}_B\mathfrak{M} \to {}_A^{\mathcal{H}}\mathfrak{M}$ is an equivalence and the inclusion $B \to A$ splits in $\mathfrak{M}_B$.*

Proof. (a) ⇒ (c) In terms of the canonical map (A.12), introduce the index notation $h^{\{1\}} \otimes_B h^{\{2\}} := \mathrm{can}^{-1}(1_A \otimes_R h)$ for $h \in H$ (implicit summation is understood). Using a right B-module retraction p of the inclusion $B \to A$, a normalized right $\mathcal{H}$-comodule map is given by $j : H \to A$, $h \mapsto p(h^{\{1\}})h^{\{2\}}$.

(b) ⇒ (c) is proven symmetrically.

(c) ⇒ (a) By Lemma 19, the lifted canonical map is a split epimorphism in $\mathfrak{M}_A^{\mathcal{H}}$. Then it is in particular a split epimorphism in $\mathfrak{M}_A^{\mathcal{H}_R}$. Since the equivalent conditions in Theorem 14 hold, A is a relative injective left $\mathcal{H}_R$-comodule. Claim (a) follows then in the same way as in the proof of Theorem 5.7. Implication (c) ⇒ (b) is proven symmetrically.

(c) ⇒ (d) The unit of the adjunction (6) is a natural isomorphism by the same reasoning as in the proof of Theorem 5.7. Then the unit of the adjunction in Proposition 11 is a natural isomorphism by Proposition 12. The counit of the adjunction in Proposition 11 is proven to be a natural isomorphism by the same steps as in the proof of Theorem 5.7.

(d) ⇒ (a) One checks that $H \otimes_R A$ is an object of ${}_A^{\mathcal{H}}\mathfrak{M}$, with A-action

$$a'(h \otimes_R a) = hS^{-1}\big(a'_{[1]}\big) \otimes_R a'_{[0]}a,$$

where $a \mapsto a_{[0]} \otimes_L a_{[1]}$ denotes the $\mathcal{H}_L$-coaction on A, and $\mathcal{H}_L$ and $\mathcal{H}_R$-coactions induced by the respective coproducts. Then the same proof of Theorem 5.7 can be applied. □

A symmetrical form of Theorem 20 is obtained by applying it to the co-opposite Hopf algebroid.

Corrections to Theorem 5.8

Theorem 5.8 is an application of Theorem 5.7 to the particular case of a *coseparable* Hopf algebroid. Its corrected proof relies on the following result.

Proposition 21. *Let $\mathcal{H}$ be a Hopf algebroid whose constituent R-coring (equivalently, the constituent L-coring) is coseparable. Then the forgetful functors $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$ and $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_L}$ are strict monoidal isomorphisms.*

Proof. The forgetful functors $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$ and $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_L}$ are strict monoidal by Theorem 7. For any right $\mathcal{H}_R$-comodule (M, ϱ_R), the equalizer (3) is $H \otimes_L H$-pure by Example 2. Symmetrically, also the purity conditions in Theorem 6(2) hold. By Theorem 6(3), this proves that the forgetful functors $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_R}$ and $\mathfrak{M}^{\mathcal{H}} \to \mathfrak{M}^{\mathcal{H}_L}$ are isomorphisms. $\square$

In light of Proposition 21, the forgetful functor $\mathfrak{M}^{\mathcal{H}} \cong \mathfrak{M}^{\mathcal{H}_R} \to \mathfrak{M}_R$ is separable. By this reasoning Theorem 5.8 holds with an unmodified proof.

Corrections to Corollary 6.6

In Section 6 we investigated equivariant projectivity of Galois extensions by a Hopf algebroid $\mathcal{H}$. Since now we have to distinguish between comodules of $\mathcal{H}$ and comodules of its constituent bialgebroids $\mathcal{H}_L$ and $\mathcal{H}_R$, we also have to distinguish between $\mathcal{H}_L$ and $\mathcal{H}_R$-equivariant projectivity in Definition 6.1 and $\mathcal{H}$-equivariant projectivity introduced below.

Definition 22. Consider a Hopf algebroid $\mathcal{H}$ and a T-ring B. A left B-module and right $\mathcal{H}$-comodule V, such that the left B-action on V is a right $\mathcal{H}$-comodule map, is said to be *T-relative $\mathcal{H}$-equivariantly projective* if the action $B \otimes_T V \to V$ is an epimorphism split by a left B-linear and right $\mathcal{H}$-colinear map.

All the following statements are proven in the same way it was done in Section 6. Proposition 6.2 needs to be replaced with

Proposition 23. *For a Hopf algebroid $\mathcal{H}$ and a right $\mathcal{H}$-comodule algebra A, set $B := A^{co\,\mathcal{H}_R}$. If the equivalent conditions in Theorem* 14 *hold then the $\mathcal{H}_L$-coaction on A possesses a left B-linear and right $\mathcal{H}$-colinear retraction.*

Theorem 6.3 needs to be replaced with

Theorem 24. *For a Hopf algebroid $\mathcal{H}$ and a right $\mathcal{H}$-comodule algebra A, take an algebra T such that $B := A^{co\,\mathcal{H}_R}$ is a T-ring. If the right $\mathcal{H}_L$-coaction on A possesses a left B-linear and right $\mathcal{H}$-colinear retraction then the following assertions are equivalent.*

(a) *A is a T-relative $\mathcal{H}$-equivariantly projective left B-module.*
(b) *A is a T-relative L-equivariantly projective left B-module.*

Proposition 6.4 needs to be replaced with

Proposition 25. *For a Hopf algebroid $\mathcal{H}$ with a bijective antipode and a right $\mathcal{H}$-comodule algebra A, take an algebra T such that $B := A^{co\,\mathcal{H}_R}$ is a T-ring. Assume that the lifted canonical map* (5.3) *is a split epimorphism of L-L bimodules (with respect to the L-actions* (5.4) *and* (5.5)*). If the equivalent conditions in Theorem* 14 *hold then A is a T-relative L-equivariantly projective right B-module, where the left L-action on A is given by* (6.1)*.*

Corollary 6.5 is substituted by

Corollary 26. *In the setting of Proposition* 25*, A is a T-relative L-equivariantly projective left B-module.*

Corollary 6.6 needs to be replaced by

Corollary 27. *For a Hopf algebroid $\mathcal{H}$ with a bijective antipode and a right $\mathcal{H}$-comodule algebra A, take an algebra T such that $B := A^{co\,\mathcal{H}_R}$ is a T-ring. Assume that the lifted canonical map* (5.3) *is a split epimorphism of L-L bimodules. If the equivalent conditions in Theorem* 14 *hold then A is a T-relative $\mathcal{H}$-equivariantly projective left and right B-module. Moreover, in this case $B \subseteq A$ is a right $\mathcal{H}_R$-Galois extension.*

Example 6.7 requires no modification if a comodule algebra of a Hopf algebroid is meant in the sense of Definition 8.

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